OSCILLATING FRACTURE PATHS IN THIN BRITTLE SHEETS: WHEN GEOMETRY RULES CRACK PROPAGATION

B.AUDOLY[1], P.M.REIS[2], B.ROMAN[3]
[1]Laboratoire de Modélisation en Mécanique, UMR 7607 CNRS/UPMC, 4 place Jussieu, case 162, 75005, France
[2]Manchester Center for Nonlinear Dynamics, Dept. physics and astronomy, University of Manchester, M139PL UK
[3]Physique et Modélisation des Systèmes Hétérogènes, UMR 7636 CNRS/ESPCI, 1O rue Vauquelin 75005 Paris, France

ABSTRACT

We report a novel mode of quasi-static oscillatory crack propagation when a cutting tip of moderately large width is driven through a thin brittle polymer film (Roman [1] , Ghatak [2]). Experiments show that the amplitude and wavelength of the oscillatory crack paths scale linearly with the width of the cutting tip over a wide range of length scales but are independent of the width of the sheet and of the cutting speed. We propose a mechanism for this instability, based on the coupling between crack propagation and out-of-plane deformations of the film. This simple model (Audoly [3]), based on classical Griffith theory, shows that the propagation of crack in brittle thin sheets follows geometrical laws.

1. INTRODUCTION

An interesting problem in fracture theory concerns the direction of propagation of the crack tip, and its instabilities: when a glass breaks, can the shape of the resulting pieces be predicted?
Recent well controlled experiments have yield a variety of interesting behaviour that is a challenge to existing theoretical formulations. An oscillatory instability in dynamic cracks was recently observed in a pre-tensioned thin rubber sheet Deegan [4] whose mechanism is still unclear. Another example is the controllable quasi-static propagation of oscillatory cracks in a thin strip of glass submitted to a thermal field (Yuse [5], Ronsin [6] Deegan [7]) which, despite its apparent simplicity, has been stimulating many studies (Adda-Bedia [8], Yang [9]).
We recently reported results on oscillatory fracture paths in a new experimental context: an object, which we denote by *cutting tip*, is perpendicularly driven through a thin polymer sheet held along its lateral boundaries, and progressively cuts the material as it advances. For large enough cutting tip widths, the crack follows a well defined and highly reproducible oscillatory path that spans a wide range of length scales, as shown in the two examples presented in Fig.1. In fact, even doing the experiment by hand yields surprisingly regular patterns.
In this communication we first recall those recent experimental results, and present a simplified geometrical model for crack propagation in thin brittle sheet. These ideas should have practical applications since the precise cutting of brittle thin sheets is common in industrial manufacturing.

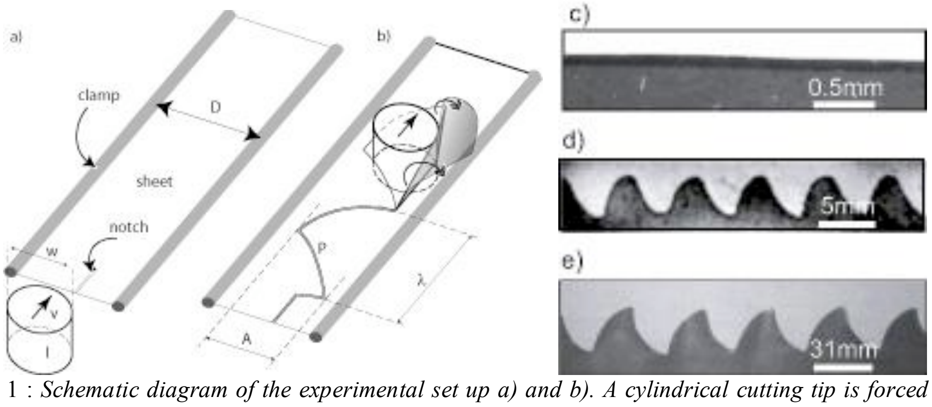

Figure 1 : *Schematic diagram of the experimental set up a) and b). A cylindrical cutting tip is forced into a clamped thin polymer sheet with a notch, leading to an oscillatory crack path P. a) Initial configuration of the experiment. b) Typical configuration during fracture. The advance of the cutting tip through the sheet leads to out of plane deformations (double arrows and region in grey). c--e) Edge of the sheet seen from above (polypropylene 27µm thick) for three cutting tip widths: c) w=0.15mm (straight path), d) w=5mm (oscillatory path), e) w=31mm (oscillatory path).*

2. EXPERIMENTAL SET-UP AND RESULTS

The experimental setup was described in detail elsewhere (Roman [1]). A schematic diagram of the apparatus is presented in Fig.1-a. A thin flat sheet is clamped along its lateral boundaries and mounted on a linear translation stage. This stage was driven at constant speed, v, towards a fixed object, the *cutting tip*, which could have either rectangular or cylindrical profile, with a variety of widths (0.05mm<w<60mm). A camera was mounted directly above the apparatus such that the propagating crack was imaged in the cutting tip's frame of reference. The sheet was initially prepared with a notch on one of its side boundaries to position and initiate the crack. Both bi-oriented polypropylene and cellulose acetate thin sheets were investigated, with thicknesses ranging between 25 and 130μm (sheet's Young's modulus E=1-2 GPa and fracture energy Γ=2-5kJ/m^2). Although polymeric, these materials are brittle since they have been severely stretched when processed into thin sheets. They undergo minimal plastic deformation during fracture propagation but, being thin, can sustain large bending without crack initiation. This explains why they are widely used in the packaging industry (resistant but easy to tear once a notch is started). The oscillatory paths discussed below were not observed in ductile materials.

As the thin sheet is forced through the fixed tip, the material is cut, leaving behind a well defined and highly reproducible path. For large enough cutting tips, the resulting path is oscillatory, two examples of which, for significantly different sizes of the cutting tip, are shown in Fig. 1d) and e). In this oscillatory regime, the non-sinusoidal oscillatory paths resembles a series of shark blades; the fracture path is made up of smooth curves connected by sharp kinks. However, and as one would expect, for thin enough objects, the path left behind the cutting tip is straight; Fig 1c). Our results point to a new instability in the fracture of thin polymer films from straight to oscillatory patterns, as the size of the cutting tip is increased.

In this communication, we mainly focus on the regime well above threshold. We found that the oscillations were essentially independent of the thickness of the sheet, of its material, of its width D, and of the speed of the experiment (Roman [1], Ghatak [2]). However both amplitude and wavelength vary linearly with the width of the cutting tip w, the only relevant parameter. These remarkable features are consistent with the robustness of the phenomenon (one can obtain very regular pattern driving a key in a plastic package), but are very surprising. Ghatak [2] suggested that the crack path resemble arches of cycloid, which is questionable as it was not supported by any physical argument. The purpose of this paper is to present a simple model for crack propagation in thin brittle sheets, based on classical Griffith theory, which thus provides the first explanation of this phenomenon.

3. A GEOMETRICAL MODEL FOR PROPAGATION

The problem of propagation of crack in thin sheets with large out-of-plane deformations is very difficult as it asks for coupling fracture theory and elasticity of thin sheets. We propose here a simplified approach. Following Griffith [10], the fracture propagates when the elastic energy release rate overcomes the fracture energy Γ. For thin sheet, elastic energy is divided in bending energy (associated with curvature) and stretching (in-plane deformation) Pogorelov [11]. For large out-of-plane deformation of thin sheets, bending is negligible compared to stretching energy: thin sheets are easy to bend. We will suppose here that bending energy is zero (infinitely thin sheets), which is a good approximation in the regime where cutting tip dimensios are large compared to sheet's thickness.

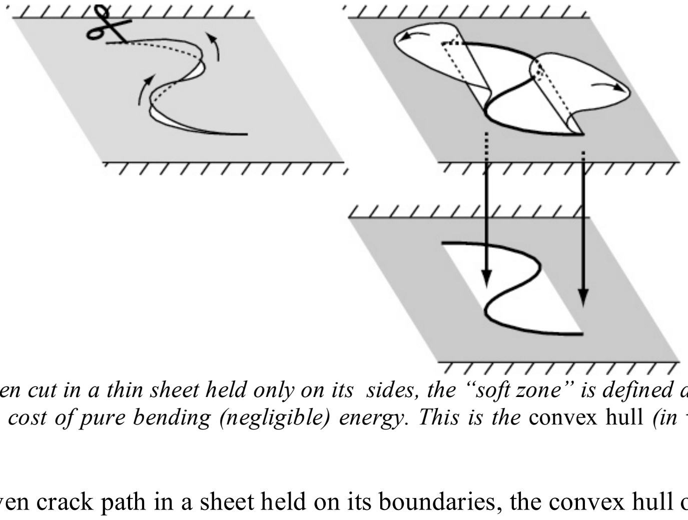

Figure 2: *For a given cut in a thin sheet held only on its sides, the "soft zone" is defined as the area that can be deformed at the cost of pure bending (negligible) energy. This is the* convex hull *(in white) of the crack path.*

Considering a given crack path in a sheet held on its boundaries, the convex hull of the path defines a zone with a remarkable mechanical property : within it, it is possible to bend the sheet without stretching it (Figure 2). However, by definition, all points outside of this zone are part of an uninterrupted line joining on the sheet the fixed boundary conditions, and is therefore materially held to the fixed boundary conditions: any displacement of these points leads to in-plane extension.

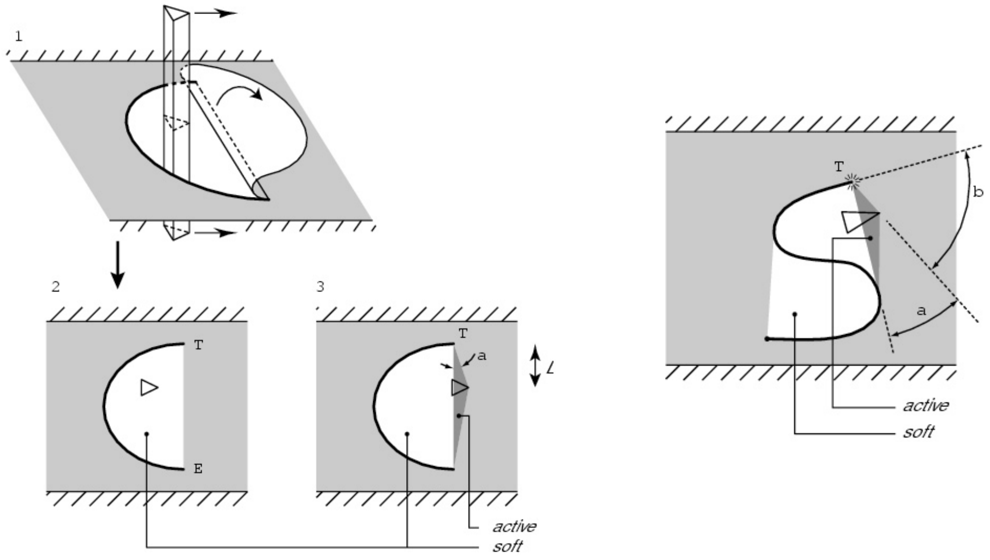

Figure 3: *When the cutting tip (here with a triangular section) in 1 and 2 is displaced outside the "soft zone" (in white) on drawing 3, tensions are generated in the "active zone"(dark grey). Elastic energy and orientation of the stress field are estimated, leading to a propagation rule based on Griffith's criterion and a simplified version of the principle of local symetry.*

When the cutting tip stays in the soft zone, the sheet easily bends away (figure3-1). However, when the tip goes beyond this soft zone, tensions appear, which eventually trigger crack propagation. On figure3, we define α as the angle, seen from the fracture tip, that measures the penetration of the cutting tip out of the soft zone. Estimating the stretching energy leads, for small angles α_c, to writing Griffith's criterion for propagation as a critical angle

$$\alpha_c \sim (\Gamma/E\,L)^{1/4}, \qquad (1)$$

where E is the Young's modulus, Γ the fracture energy, and L a typical lengthscale (distance from the cutting tip to the fracture tip).
To predict the direction of crack propagation, one generally uses the principle of local symmetry. This phenomenological criterion states that the cracks propagates preferentially in mode I, and requires the computation of the stress intensity factors. In our simplified model, we observe that the opening stresses are distributed along the direction α from the tip. We will thus assume that propagation takes place in a direction with an angle β with respect to the line joining cutting tip-crack tip, β being close to $\pi/2$.

Using the specific mechanics of thin sheets, the propagation of a crack is thus computed through *geometrical rules* only. The mechanical properties of the material are hidden in the parameters α_c and β. Note also that there is no *ad hoc* ingredient for oscillations; this simplified model is very general and should describe all situations where thin brittle sheet undergo large out of plane deformation together with fracture.

4. COMPARISON WITH EXPERIMENTS

The numerical integration of the propagation model is implemented as follows. Starting from a given crack path, we compute the soft zone. The cutting tool is advanced of a step. It it lies out of the soft zone with an angle $\alpha > \alpha_c$, propagation of the crack is triggered in direction β. With the crack, the soft zone advances, and reduces the new angle α, up to the point where we are back to α_c, and the crack stops. We now are back to the first step, where the cutting tool can be moved again.

The numerical intergation of this geometrical model shows that it generates oscillations for a wide range of parameters, and initial conditions, with crack path fitting extremely well the experiments

(Figure 4), for α_c =? β=? (only adjustable parameters). Note that the crude estimate (1) gives the right order of magnitude for α_c and β is close to $\pi/2$, as expected.

Moreover, the model reproduces the temporal evolution of the crack, with quasistatic phases, followed with a sudden direction change and a dynamic jump to a new position. On Figure 4 is represented the temporal evolution of the x and y position of the crack, wher the large jumps in x are clearly visible. These dynamic jumps observed in the experiment have thus a geometrical origin (Audoly [3, 12]).

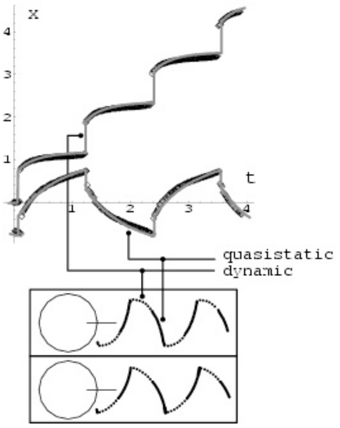

Figure 4: *top : Experimental (points) and model prediction (continuous line) for crack position as a function of time. Bottom : Experimental and numerical path, for the same cylindrical cutting tip (circles). In thick line are presented path with quasistatic propagation; points in light grey correspond to dynamic propagation.*

This geometrical model explains why the pattern is independent (Roman [1]) of the width of the sheet : the propagation rules are here defined in terms of geometry of the crack path only. This, in turn, comes from particular properties of thin sheets mechanics. Since the pattern amplitude and wavelength have a weak on the parameter (α_c ,β), and that α_c itself has a very weak dependance on scale and mechanical properties (exponent ¼), this model predicts that the pattern should at first order scale with the cutting tip size, and independantly of material and thickness. All these features were observed in the experiments (Roman [1]). Put together, these properties explain why the phenomenon is so robust, and can be observed even when perfoming the experiment at hand. The instability mechanism is also revealed by the model. Because the soft zone does not transmit stresses, and can be formally removed from the sheet, the cutting tip only pushes on one side of the crack at a rim. Suppose it acts on the rigth rim, because of the angle α_c being small and β being of the order of $\pi/2$, the crack propagates to the left, leading thus to an instability of the straight path.

5. CONCLUSION

We presented a model (based on Griffith's criterion) for the propagation of cracks in brittle thin sheets. A remarkable feature of the model is that the crack path can be predicted using *purely geometrical rules* for propagation. This is due to the particular mechanical properties of thin sheets, which deformations are strongly linked with geometry.

We have shown that the model captures the mechanism of a new crack path instability observed as a blunt cutting tip (with radius much larger than the sheet's thickness) is pusehd through a brittle thin sheet.

Our model is not built for this specific situation and should describe more general propagation of cracks in thin brittle sheets.

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